

AS Level Further Mathematics A

Unit Y535/01 Additional Pure Mathematics

Tuesday 22 May 2018 – Afternoon

Time allowed: 1 hour 15 minutes



You must have:

- Printed Answer Booklet
- Formulae AS Level Further Mathematics A

You may use:

- a scientific or graphical calculator

INSTRUCTIONS

- Use black ink. HB pencil may be used for graphs and diagrams only.
- Complete the boxes provided on the Printed Answer Booklet with your name, centre number and candidate number.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** If additional space is required, you should use the lined page(s) at the end of the Printed Answer Booklet. The question number(s) must be clearly shown.
- Do **not** write in the barcodes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g \text{ m s}^{-2}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION

- The total mark for this paper is **60**.
- The marks for each question are shown in brackets [].
- **You are reminded of the need for clear presentation in your answers.**
- The Printed Answer Booklet consists of **12** pages. The Question Paper consists of **4** pages.

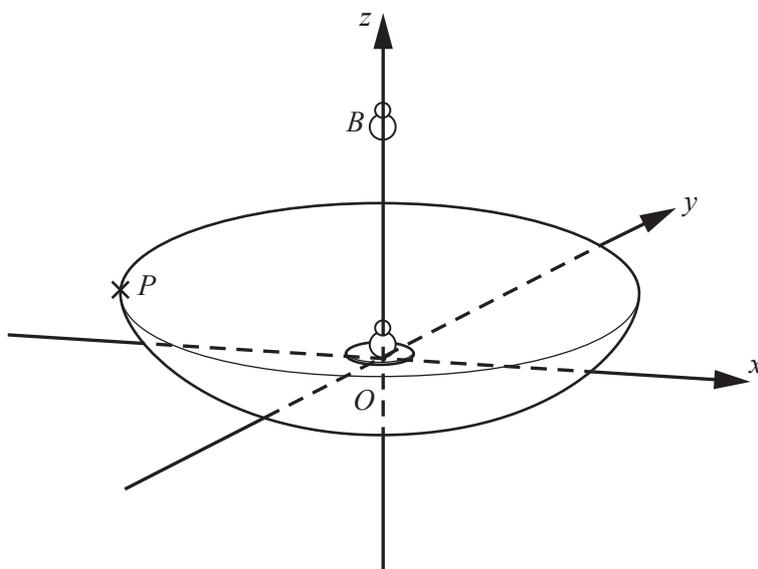
Answer **all** the questions.

- 1 The points A , B and C have position vectors $6\mathbf{i} + 2\mathbf{j} + 4\mathbf{k}$, $13\mathbf{i} + 2\mathbf{j} + 5\mathbf{k}$ and $16\mathbf{i} + 6\mathbf{j} + 3\mathbf{k}$ respectively.
- (i) Using the vector product, calculate the area of triangle ABC . [5]
- (ii) Hence find, in simplest surd form, the perpendicular distance from C to the line through A and B . [3]
- 2 The surface with equation $z = 6x^3 + \frac{1}{9}y^2 + x^2y$ has two stationary points.
- (i) Verify that one of these stationary points is at the origin. [4]
- (ii) Find the coordinates of the second stationary point. [5]
- 3 Given that n is a positive integer, show that the numbers $(4n + 1)$ and $(6n + 1)$ are co-prime. [3]
- 4 The group G consists of a set of six matrices under matrix multiplication. Two of the elements of G are
- $$\mathbf{A} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \text{ and } \mathbf{B} = \begin{pmatrix} 1 & -1 \\ 0 & -1 \end{pmatrix}.$$
- (i) Determine each of the following:
- \mathbf{A}^2
 - \mathbf{B}^2
- [2]
- (ii) Determine all the elements of G . [4]
- (iii) State the order of each non-identity element of G . [3]
- (iv) State, with justification, whether G is
- abelian
 - cyclic.
- [2]
- 5 For integers a and b , with $a \geq 0$ and $0 \leq b \leq 99$, the numbers M and N are such that
- $$M = 100a + b \text{ and } N = a - 9b.$$
- (i) By considering the number $M + 2N$, show that $17 \mid M$ if and only if $17 \mid N$. [4]
- (ii) Demonstrate step-by-step how an algorithm based on the result of part (i) can be used to show that 2058376813901 is a multiple of 17. [4]
- 6 The Fibonacci sequence $\{F_n\}$ is defined by $F_0 = 0$, $F_1 = 1$ and $F_n = F_{n-1} + F_{n-2}$ for all $n \geq 2$.
- (i) Show that $F_{n+5} = 5F_{n+1} + 3F_n$ [3]
- (ii) Prove that F_n is a multiple of 5 when n is a multiple of 5. [5]

- 7 The ‘parabolic’ TV satellite dish in the diagram can be modelled by the surface generated by the rotation of part of a parabola around a vertical z -axis. The model is represented by part of the surface with equation $z = f(x, y)$ and O is on the surface.

The point P is on the rim of the dish and directly above the x -axis.

The object, B , modelled as a point on the z -axis is the receiving box which collects the TV signals reflected by the dish.



- (i) The horizontal plane Π_1 , containing the point P , intersects the surface of the model in a contour of the surface.
- (a) Sketch this contour in the Printed Answer Booklet. [1]
- (b) State a suitable equation for this contour. [1]
- (ii) A second plane, Π_2 , containing both P and the z -axis, intersects the surface of the model in a section of the surface.
- (a) Sketch this section in the Printed Answer Booklet. [1]
- (b) State a suitable equation for this section. [1]
- (iii) A proposed equation for the surface is $z = ax^2 + by^2$. What can you say about the constants a and b within this equation? Justify your answers. [3]
- (iv) The real TV satellite dish has the following measurements (in metres): the height of P above O is 0.065 and the perimeter of the rim is 2.652. Using this information, calculate correct to three decimal places the values of
- a and b ,
 - any other constants stated within the answers to parts (i)(b) and (ii)(b). [4]
- (v) Incoming satellite signals arrive at the dish in linear ‘beams’ travelling parallel to the z -axis. They are then ‘bounced’ off the dish to the receiving box at B .
- On the diagram for part (ii)(a) in the Printed Answer Booklet draw some of these beams and mark B .
 - If the values of a and b were changed, what would happen? [2]

END OF QUESTION PAPER

OCR

Oxford Cambridge and RSA

Copyright Information

OCR is committed to seeking permission to reproduce all third-party content that it uses in its assessment materials. OCR has attempted to identify and contact all copyright holders whose work is used in this paper. To avoid the issue of disclosure of answer-related information to candidates, all copyright acknowledgements are reproduced in the OCR Copyright Acknowledgements Booklet. This is produced for each series of examinations and is freely available to download from our public website (www.ocr.org.uk) after the live examination series.

If OCR has unwittingly failed to correctly acknowledge or clear any third-party content in this assessment material, OCR will be happy to correct its mistake at the earliest possible opportunity.

For queries or further information please contact the Copyright Team, First Floor, 9 Hills Road, Cambridge CB2 1GE.

OCR is part of the Cambridge Assessment Group; Cambridge Assessment is the brand name of University of Cambridge Local Examinations Syndicate (UCLES), which is itself a department of the University of Cambridge.